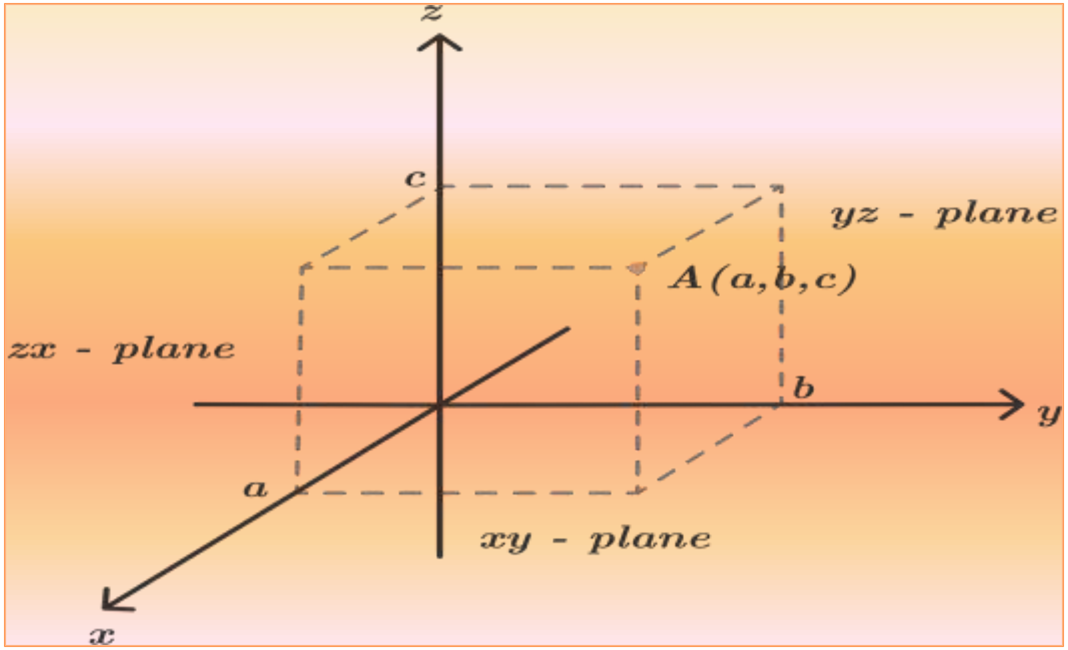
THREE DIMENSIONAL GEOMETRY
PLANES - I



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OBJECTIVES

- To understand the definition of the Plane
- To know the different types of Planes
- To know the method of finding the equation of the plane passing through three points
- To know the method of finding the angle between two planes
- To understand the distance between two parallel planes

LEARNING OUTCOMES

- Student can be able to find the equation of the plane through the given conditions
- Student can be able to find the angle between the planes
- Student can be able to find the distance between two parallel planes

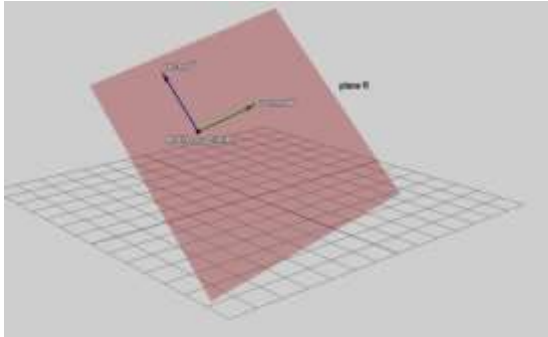
- Def. : A surface is called a Plane if all the points of a straight line joining any two points on the surface lie on it.

- GENERAL FORM OF THE PLANE

  The first degree general equation in x, y, z represents a plane.

  i.e. The equation ax+by+cz+d=0 is referred as general equation of the plane. Where a, b, c, are the dr's of the normal to the plane.

- NORMAL FORM OF THE PLANE



- The vector equation of the plane which is at a distance of p from the origin along the unit vector **n** is **r.n =** p
- The equation $lx + my + nz = p$ is a normal form of the plane where p is a positive real number and

  $l^2 + m^2 + n^2 = 1$

PERPENDICULAR DISTANCE

The perpendicular distance from the point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz = d$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

PLANE PASSING THROUGH A POINT

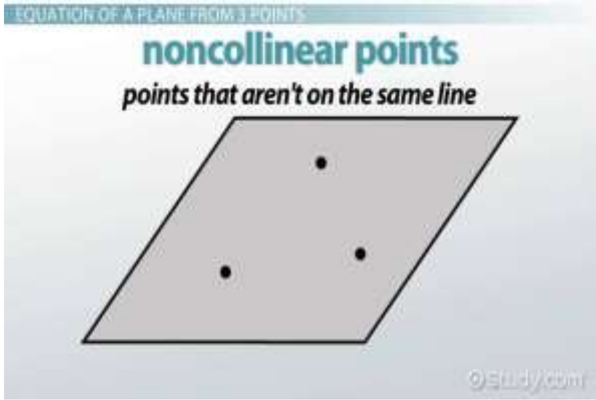
Equation of the plane passing through a point $P(x_1, y_1, z_1)$ is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

PLANE PASSING THROUGH THREE POINTS

Equation of the plane passing through three non collinear points (x_1, y_1, z_1) (x_2, y_2, z_2) (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$



EX.. Find the equation of the plane passing through the points $(1,2,1),(1,1,0),(-2,2,-1)$

Sol. METHOD I

Given points $(x_1,y_1,z_1) = (1,2,1)$

$(x_2,y_2,z_2) = (1,1,0)$

$(x_3,y_3,z_3) = (-2,2,-1)$

Equation of the plane passing through three points is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-1 \\ 1-1 & 1-2 & 0-1 \\ -2-1 & 2-2 & -1-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-1 \\ 0 & -1 & -1 \\ -3 & 0 & -2 \end{vmatrix} = 0$$

$$=> \quad 2(x-1)+3(y-2)-3(z-1)=0$$

$$=> \quad 2x+3y-3z-5=0$$

is the required equation of the plane.

METHOD II

equation of the plane passing through the point (1, 2, 1) is

a (x-1) + b (y-2) + c (z-1) = 0 ----------------------------(1)

Since the plane (1) passing through the point (1, 1, 0)

a (1-1) + b (1-2) + c (0-1) = 0

⇒ 0.a – b –c =0 ------------------------------(2)

Since the plane (1) passing through the point (-2, 2, -1)

a (-2-1) + b (2-2) + c (-1-1) = 0

⇒ -3a + 0.b - 2c =0 -------------------------------(3)

Eliminating a, b, c from the equations (1), (2) and (3) we get

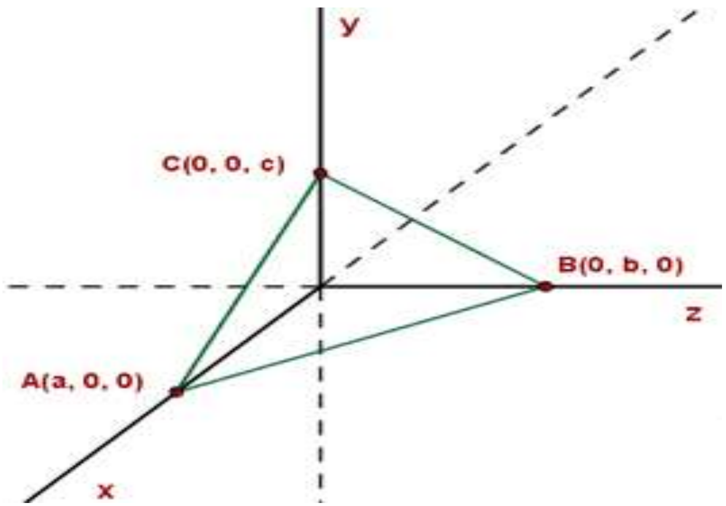
$$=> \begin{vmatrix} x-1 & y-2 & z-1 \\ 0 & -1 & -1 \\ -3 & 0 & -2 \end{vmatrix} = 0$$

$$=> \quad 2x+3y-3z-5=0$$ is the required equation of the plane

INTERCEPT FORM OF THE PLANE

The equation of the plane having x-intercept a, y-intercept b, z-intercept c is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$



It is also called as the intercept form of the plane.

ANGLE BETWEEN TWO PLANES

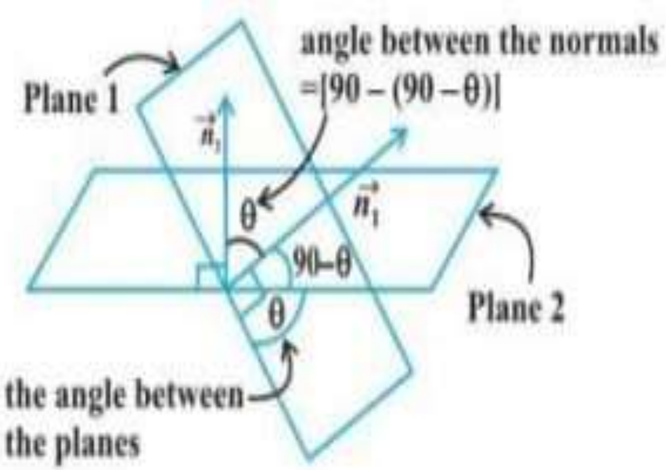
The angle between the normals of two planes is called the angle between the planes.

If θ is the angle between the planes

$a_1x+b_1y+c_1z+d_1=0$, $a_2x+b_2y+c_2z+d_2=0$

Then

$$cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$



If the planes are parallel <=> $a_1:b_1:c_1=a_2:b_2:c_2$

If the planes are perpendicular <=> $a_1a_2+b_1b_2+c_1c_2=0$

EX.. Find the angle between the planes $2x - y + z = 6\,, x + y + 2z = 7$

- Sol.. If θ is the angle between the given planes , then

$$\cos\theta = \frac{(2)(1) + (-1)(1) + (1)(2)}{\sqrt{4+1+1}\sqrt{1+1+4}}$$

$$=> \cos\theta = \frac{1}{2}$$

$$=> \theta = 60^{\circ}$$

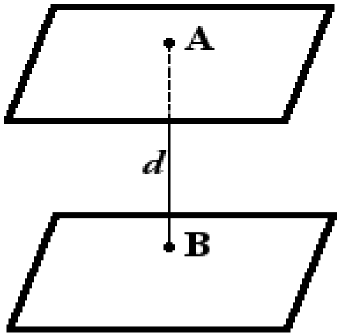
Therefore angle between the given planes is 60°

- If the planes are parallel then their normals are also parallel
- If the planes are perpendicular then their normals are also perpendicular
- The equation of a plane parallel to ax + by + cz + d=0 is ax + by + cz + k=0

DISTANCE BETWEEN TWO PARALLEL PLANES

The distance between two parallel planes $ax + by + cz + d_1 = 0$

$ax + by + cz + d_2 = 0$ is $\dfrac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$



EX ... Find the distance between the parallel planes $12x - 3y + 4z - 7 = 0$, $12x - 3y + 4z + 6 = 0$

Sol.. Distance between the parallel planes = $\dfrac{|-7 - 6|}{\sqrt{12^2 + (-3)^2 + 4^2}}$

$$= \frac{13}{13} = 1$$

---------- THANK YOU -----------